

RAIN RATE MODEL AND RAIN CELL SIZE BASED ON 18-YEAR  
RAIN DATA FOR RADIOWAVE PROPAGATION IN SAUDI ARABIA

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The propagation of electromagnetic waves in the atmosphere involves the absorption and scattering by hydrometers and dust particles. At extremely high frequency (EHF) rain attenuation sets the limit on hop length at rain regions with moderate or heavy rain. The situation is different, however, in arid land, where the hop length could be further extended, depending on rain rate (mm/hr) distribution and rain cell size (km), along the hop path.

The aim of the present paper is to determine an empirical model for rain rate distribution throughout the Kingdom of Saudi Arabia as an example of arid land, as well as rain cell size(s) for various rain regions of Saudi Arabia as a function of instantaneous rain rate, based on 18-year meteorological data.

### I. INTRODUCTION

The role of rain profiles in determining the statistics of attenuation along radio paths of frequencies above 10 GHz is a matter of large interest. Even though there is a world-wide agreement in using proper descriptive parameters such as "effective rain intensity" and "effective rainy path length", no definite procedures to measure, work out and statistically describe such parameters have been accepted so far [8]. A new approach here suggested aims at processing the long-term measurements of point rain rate available, for the period 1963-1980 in Saudi Arabia, from the Ministry of Agriculture and Water (MAW) [1] to predict the rain rate profile.

The availability of rain data from various recording stations separated by few kilometers to several tens of kilometers within one area in Saudi Arabia seems to be of great importance in estimating the spatial distribution of point rain rate. Such a parameter will be obtained by correlating data obtained from various stations for the same rain event.

Moreover, an identification of the rain cell size is to be attempted based on the rain profile and owing to the fact that heavy downpour rain is cellular in nature and has limited coverage, on the other hand, light rainfall is usually widespread in character [1].

### II. RAIN DATA FOR SAUDI ARABIA

#### 2.1 The Data Source

The available rainfall intensity data consist of records from 137 recording rainfall stations for the period 1963-1980. These data represent the intensity portion of the rainfall data collected by the Hydrology Division, Ministry of Agriculture and Water [1].

The available data consist of about 16300 events. Individual event data consist of maximum cumulative rainfall in mm of rain that occurred during measuring periods of (10, 20, 30, 60, 180, 360 and 720 minutes) in addition to total rainfall, duration, type of event and time at which event occurred.

#### 2.2 Utilization of Rain Data

For radio path engineering a procedure is needed to calculate the rain attenuation distributions on millimetric radio path using the available rainfall data and since the attenuation is related to rain rate in (mm/hr) rather than rainfall intensity in (mm) hence, it is necessary to transfer the available data into rain rate.

Upon examining the data for the past 18 years it was found that there are missing data due to many reasons such as (1) equipment failure, (2) human error in picking up rainfall precipitation from a continuous rain gauge chart (3) data entry errors. It has been found that rainfall rate distribution could be well approximated by a log-normal law [3,4,5]. Hence, the missing data can be estimated before processing the entire data set. Missing data have been calculated using the least square method to find the best curve which represents the measured data.

Once curve fitting is completed a consistency check was made to ensure reliability of the point rain rate distribution. Both curve fitting and consistency tests were done using Amdahl 5840 computer.

### III. PREDICTION OF RAIN RATE PROFILE

#### 3.1 Rain Rate Model

Rain attenuation over a hop is computed by integrating the specific attenuation over the path length: the rain intensity profile along the path is therefore needed. Due to the non-uniformity of this profile, the value of rain attenuation exceeded for a certain percentage of the year increases in less than linear proportion with path length [3,4].

Consequently, cumulative distribution of rain attenuation at a given frequency and polarization differs from one region to another only because the point cumulative distributions of rainfall intensity assumed for the two regions are different. The entire prediction procedure is therefore based on the rainfall intensity data which are usually acquired, for a certain region, at one or several points within that region [3].

#### 3.2 Prediction Procedures

Data used in the prediction method of rain rate profile along the path are obtained from the South-West region in Saudi Arabia, namely ABHA sub-region and TAIF sub-region.

Twenty three rainfall recording stations have been selected from both sub-regions such that distances between stations of the same region vary between four and forty kilometers. Nine stations are selected from ABHA with a total number of (2839) events, and fourteen stations from TAIF with a total number of (1796) events.

The prediction procedure can be summarized as follows:

(1) Search for common rainy days between stations of the same region. (In the given data, each rainy day is identified by the year, month, and day on which it occurs, so common rainy days are those events which occur on the same time at different locations).

(2) Define the mixed event as that event which consists of the maximum 10-minute rain rates selected from each event found in a set of common events. Thus, one mixed event is obtained for each set of common events.

(3) Arrange the rates in each mixed event in a descending order such that  $R(A) > R(B) > R(C) > R(1)$  where  $R(1)$  is the maximum 10-minute rain rate at station 1. Hence, station A becomes the center of the rain cell with rate  $R(A)$  mm/hr. Similarly  $R(B)$  and  $R(C)$  are rain rates at distances AB kilometers and AC kilometers, from the center, respectively.

(4) Define  $R_E$  as the rain rate exceeded, measured in steps as 1,5,10, ..., 100 mm/hr.

(5) Select from each mixed event the maximum rate,  $R_{max}$ , and compare it with the nearest value of  $R_E$  until  $R_{max} > R_E$ . Repeat steps (3) and (5) for each mixed event.

(6) Mixed events having their maximum rain rates exceeding the same value of  $R_E$  are averaged to get the average distribution of the maximum rain rate along the path. This is done by fitting the data in each mixed event (i.e. rain rate data versus path length) using the polynomial least square method of curve fitting. Rain rate values at specified distances can then be obtained for each mixed event and hence averaging can be made for those mixed events having their maximum rates exceeding the same value of  $R_E$ .

Table 1 presents the distribution of average rain rates that exceed specified values of  $R_E$  at specified distances.

Rate Exceeded (mm/H)	Distance						
	0 Km	6 Km	12Km	18Km	24Km	30Km	36Km
100	124.0	95.0	57.7	39.5	11.4	0.0	0.0
90	94.0	79.4	64.8	50.2	35.6	21.0	6.4
80	86.2	70.4	54.6	38.9	23.1	7.4	0.0
75	77.3	59.1	40.8	22.6	4.4	0.0	0.0
70	72.2	61.3	50.5	39.6	28.8	17.9	7.0
65	67.2	54.6	42.1	29.5	16.9	4.4	0.0
60	62.4	51.3	40.3	29.2	18.2	7.1	0.0
55	58.1	46.4	34.6	22.9	11.1	0.0	0.0
50	52.9	43.2	33.5	23.8	14.1	4.4	0.0
45	47.4	38.9	30.4	21.9	13.4	4.8	0.0
40	42.2	34.0	25.8	17.6	9.3	1.1	0.0
35	37.0	29.6	22.3	14.9	7.5	0.1	0.0
30	32.7	26.5	20.4	14.2	8.0	1.9	0.0
25	27.7	22.9	18.1	13.2	8.4	3.6	0.0
20	22.5	18.9	15.4	11.8	8.2	4.6	1.0
15	17.4	14.8	12.1	9.5	6.8	4.2	1.5
10	12.6	10.9	9.2	7.5	5.8	4.2	2.5
5	7.5	6.5	5.4	4.4	3.3	2.3	1.2
1	3.4	3.2	2.9	2.7	2.5	2.2	2.0

Table 1. Average rain rate distribution along the path.

### 3.3 Nature of Rate Distribution

Three different distribution functions have been examined to suit the data in Table 1 using the least square method:

(1) The first function is the normal distribution of the form:

$$R(x) = a e^{bx^2} \quad (1)$$

where  $R$  is rain rate in mm/hr,  $x$  is the distance in kilometer,  $a$  and  $b$  are constants.

(2) The second function is the exponential distribution of the form

$$R(x) = a e^{bx} \quad (2)$$

(3) The third function is polynomial distribution of the fifth order:

$$R(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 \quad (3)$$

Examining the rain rate data, it has been found that  $a_0$  has the same value as the maximum rain rate of the mixed event, and coefficients  $a_2, \dots, a_5$  are very small and can be eliminated without effecting the profile. Therefore, equation (3) is approximated by the straight line:

$$R(x) = R_{max} + a_1x \quad (4)$$

Good agreement has been achieved between results of equation (4) and the original data in Table 1, while the first two functions did not agree with the original data.

Table 2 presents the resulted rain rate models for various values of rain rate exceeded.

$R(E)$ (MM/H)	$R(x)$ (MM/H)
1	3.47 - 0.0402 x
5	7.54 - 0.1735 x
10	12.61 - 0.2802 x
15	17.45 - 0.4405 x
20	22.58 - 0.5981 x
25	27.75 - 0.8031 x
30	32.74 - 1.0269 x
35	37.07 - 1.1212 x
40	42.27 - 1.3702 x
45	47.44 - 1.4181 x
50	52.90 - 1.6142 x
55	58.18 - 1.9599 x
60	62.42 - 1.8422 x
65	67.22 - 2.0925 x
70	72.22 - 1.8091 x
75	77.38 - 3.0403 x
80	86.21 - 2.6266 x
90	94.01 - 2.4309 x
100	24.12 - 4.6956 x

Table 2. Rain rate models for different values of  $R(E)$

### 3.4 The Rate Profile

It is known that heavy downpour rain is cellular in nature and has limited coverage. On the other hand, light rainfall is usually wide spread in character [2]. Thus it is expected that heavy rain rate profile along the path will be more steep than the profile of light rain rate.

Referring to the data in Table 2, it is obvious that as maximum rain rate increases, coefficient  $a_1$  also increases, hence, there is an expected relation between these two parameters.

Four different functions have been examined to find the best relation between  $R_{max}$  and  $a_1$ , these functions are: Normal Distribution, Exponential Distribution, Polynomial Distribution, and Power Law.

Satisfactory results were obtained only when using the Power Law function of the form:

$$a_1 = -12.226 \times 10^{-3} \cdot R_{max}^{1.2297} \quad (5)$$

Hence, equation (12) becomes:

$$R(x) = R_{max} - 12.226 \times 10^{-3} \cdot R_{max}^{1.2297} \cdot x \quad (6)$$

A computer program was written to perform the previously mentioned procedures for the prediction of rain rate profile.

#### IV. IDENTIFICATION OF RAIN CELL SIZE

##### 4.1 Rain Cells

High intensity rain falls within a limited zone. The intensity of rain is usually greater towards the center of the zone [6].

In CCIR Report 563-1 [10], a connection is given between rain intensity and the size of rain cells. The size was determined by measurements with radar or rain gauges and is defined as follows [6]:

"The size of a cell with a given intensity is the diameter of a circle within which the rain intensity is equal to, or greater than the given intensity."

Distribution of rain rate intensity in mm/hr as a function of distance in km was found to be linear and given as:

$$R(x) = R_{max} + mx \quad (7)$$

where,  $R_{max}$  = maximum rate at cell center, i.e., at  $x = 0$  km,  
 $m$  = negative coefficient  
 $x$  = distance in kilometer.

Two different approaches, other than the CCIR definition which will be considered later, has been used to determine the rain cell size (km), depending on the previously established rain intensity model,  $R(x)$ :

(1) A rain cell is usually surrounded by certain rain of lower intensity, called residual rain. The intensity of residual rain increases somewhat as rain intensity increases [2,5,6]. Since the size of the rain cells, in accordance with the same model, declines somewhat as rain intensity increases, it is reasonable that the results will be about the same if both magnitudes are considered constant and equal to some values to be found [7].

Assume constant uniform rain model with a constant rate of  $R_0$  mm/hr over the area covered by rain (the circular rain cell zone). Our goal now is to find the equivalent uniform rain rate intensity  $R_0$ .

Total attenuation due to rain, in dB, over the entire rain cell is given by the following equation [9]:

$$Loss = 2 \int_0^x A(x) dx \quad (8)$$

where  $A(x)$ , the excess attenuation in dB/km due to rainfall, appears at present to be well established in the form of the power law:

$$A(x) = a R^b(x) \quad (9)$$

where,  $a$  and  $b$  are constants which vary with frequency and polarization of the millimetric wave signal. Hence, equation (9) becomes:

$$Loss = 2 \int_0^x a R^b(x) dx \quad (10)$$

Now, if we assume constant (uniform; homogeneous) rainfall (mm), i.e. a constant rain rate ' $R_0$ ' (mm/hr), then equation (9) becomes:

$$A = a R_0^b \quad \text{dB/Km} \quad (11)$$

For the actual case, where rainfall is not uniform, it is important to estimate  $R_0$  such that total attenuation due to rainfall remains the same. Hence, using equation (11),

$$Loss = 2 \int_0^x a R_0^b dx \\ = 2 a R_0^b x_m \quad (12)$$

Equations (7) and (10) give,

$$Loss = 2 \int_0^x a (R_{max} + mx)^b dx \quad (13)$$

Equating (12) and (13):

$$2 a R_0^b x_m = 2 \int_0^x a (R_{max} + mx)^b dx$$

$$\text{but, } R(x_m) = R_{max} + mx_m = 0$$

and with manipulation, we get

$$R_0 = \frac{R_{max}}{(b+1)^{1/b}} \quad \text{mm/hr,} \quad (14)$$

For EHF, parameter  $b$  is approximately unity, hence the equivalent uniform rain rate ' $R_0$ ' is approximately equal half the maximum value of the given rate,  $R(x)$ , for a cell size of 2  $x_m$  kilometer, i.e.

$$R_0 = \frac{R_{max}}{2} \quad \text{mm/hr,} \quad (15)$$

Using the previously determined rain rate model (Eq. (6)), and applying the first approach in determining the rain cell size, Fig. 1 presents rain rate intensity (mm/hr) versus rain cell size (Km).

## V. CONCLUSION

Three distribution functions have been examined for the estimation of rain rate profile along the path. They are the Gaussian-shaped rain profile, the exponential-shaped rain profile, and the triangular-shaped rain profile. Analysis of the results indicates that the triangular-shaped rain rate profile provides very satisfactory agreement with the original data, and the profile is approximated by a straight line function of the given maximum rain rate and distance. The same profile was proposed in some published papers [8].

Two different approaches have been used to determine a relation between the rain rate intensity and the size of rain cells using the predetermined rain rate profile. The first approach assumes an equivalent uniform rain rate,  $R_0$  mm/hr, over the rain cell zone. The value of  $R_0$  was found as half the maximum value of the given rate,  $R(x)$ , with a cell size of  $2x$  Km. The second approach assumes a rain cell with a uniform rate equals the maximum value of  $R(x)$ . The cell radius, in this case, equals to the distance within which the given rate is dropped from  $R_{max}$  to  $\beta R_{max}$ . At present, a third approach based on the CCIR definition of rain cell size is in progress. The results will be reported as soon as they are available.

## VI. ACKNOWLEDGEMENT

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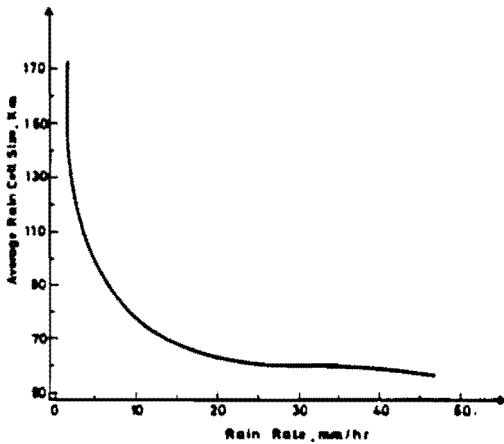


Fig. 1. Rain cell size versus rain rate (based on the 1st approach).

(2) The actual rain cell for any given rain rate intensity is represented by an equivalent rain cell of circular zone in which the rain within the zone is homogeneous with an intensity equal to the given maximum rain rate intensity [6]. The radius of the circular zone is established by assuming that it is equal to the distance where the given rain intensity dropped from  $R_m$ , at cell center, to  $\beta R_m$ , where  $0.5 < \beta < 1$ .

$$\begin{aligned} \text{Hence if } R(x) &= R_{max} + \alpha x \\ \text{then } R(r) &= R_{max} + \alpha r \end{aligned} \quad (16)$$

where,  $r$  is the rain cell radius in kilometer. Hence,

$$R(r) = \beta R_{max} \quad (17)$$

Equating equations (16) and (17) to get:

$$r = R_{max} \frac{(\beta - 1)}{\alpha} \quad \text{Km}, \quad (18)$$

The rain cell size (diameter of the cell) then becomes equal to:

$$d = \frac{2}{\alpha} (\beta - 1) R_{max} \quad \text{Km}, \quad (19)$$

Fig. 2 presents rain intensity (mm/hr) versus rain cell size (Km) curves for different values of ' $\beta$ ' based on the second approach.

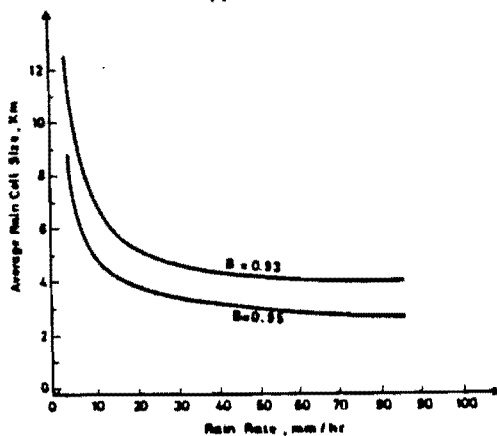


Fig. 2.