Injection-Locked Passively Q-Switched Lasers

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Abstract—The time-dependent behavior of an injection locked single-mode passively Q-switched laser may be described by three coupled first-order rate equations relating the laser gain, the absorber loss, and the optical power density. Injection locking results in a substantial increase in the mean output power and the output pulse energy and higher pulsation rate than when freely running. Stability analysis allows the determination of the maximum injected power that allows pulsations.

I. Introduction

THE problem of injection locking of a pulsed laser oscillator has become very important in recent years. Theoretical studies of this problem have been presented for TEA-CO_2 lasers and dye lasers [1]-[3]. These investigations concentrated on a pulsed pumping operation.

An alternative technique is that of passive Q-switching of the laser using an intracavity saturable absorber [4], [5]. Intracavity absorbers are simple to construct, align, and are inexpensive [6]. Buczek, Freiberg, and Skolnick reported injection locking of a passively Q-switched CO_2 laser to produce frequency-stable high-repetition rate phase-coherent pulses [7]. The locking range obtained in a pulsed oscillator is greater than that achievable with CW oscillation.

The purpose of this work is to present a detailed analysis of the effect of a CW injected optical signal on the performance of a passively Q-switched laser. In the present work the analysis of Powell and Wolga for passive Q-switching [8], and of Haus for relaxation oscillations [9], are extended to include an externally injected signal [10].

This approach enables the determination of the average output power, pulse energy, pulse repetition frequency, and also the maximum injected signal that allows pulsation.

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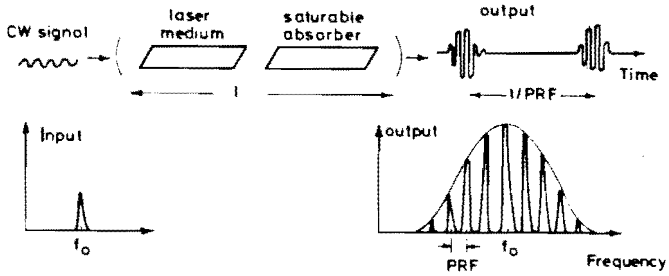

Fig. 1. Model for injection locking of a passively Q-switched laser.

II. Rate Equations

A CW optical signal $E_{in} \cos(\omega_o t + \phi)$ is injected into a variable-Q cavity of a laser which contains a saturable absorber. Under locking conditions, the electric field is assumed as $v(t) \cos \omega_o t$, where $v(t)$ is the pulse envelope which is slowly time varying compared to ω_o. Fig. 1 shows the model used for injection locking of a passively Q-switched laser. In the analysis the following assumptions are made:

1) the laser and absorber media are homogeneously broadened,

2) the time dependence of the population difference is described by a single relaxation rate,

3) all gain and loss parameters are uniform along the optical cavity.

The time-dependent behavior of a passively Q-switched laser may be described by three-coupled first-order nonlinear equations, relating the population difference of the laser medium n_l, and the negative population difference of the absorber n_a, and the photon number density S, where

$$S = \frac{1}{2} \epsilon_o |v(t)|^2 / h\omega_o. \qquad (1)$$

In the presence of an injected signal the rate equations take the form [8], [10]

$$\dot{n}_l = -2\alpha n_l S - \gamma_l (n_l - n_l^o) \tag{2}$$

$$\dot{n}_a = -2\beta n_a S - \gamma_a (n_a - n_a^o) \tag{3}$$

$$\dot{S} = (\alpha n_l - \beta n_a) S - \gamma_c S + \left(\frac{c}{l}\right) \sqrt{S S_{in}} \cos \phi \tag{4}$$

where n_l^o, n_a^o the equilibrium differences

- α, β field interaction coefficients,
- γ_l, γ_a relaxation rates for the laser medium and the absorber,
- γ_c cavity loss rate,
- c the velocity of light,
- l the optical cavity length,

and

- S_{in} the injected photon number density.

The factor 2 in (2) and (3) was not included in [8]. Within the locking range, the phase angle ϕ between the injection signal and the resultant field amplitude is a function of the difference of the injection signal frequency ω_o and the free-running "natural" oscillation ω_n, according to [10]

$$\omega_o - \omega_n = (c/2l) \sin \phi \sqrt{S_{in}/S}. \tag{5}$$

It is to be noted that outside the locking range a fourth rate equation need be postulated to account for the phase incoherent "natural" pulsed oscillation. In what follows we consider that the injection signal is tuned to produce maximum output so that the phasing $\phi = 0$.

Equations (2)-(4) may be rewritten in terms of the gain per pass g, absorber loss per pass q, and the optical power density P [9], as follows:

$$\left(\frac{1}{\gamma_l}\right)\dot{g} = -g(P/P_l) - (g - g_o) \tag{6}$$

$$\left(\frac{1}{\gamma_a}\right)\dot{q} = -q(P/P_a) - (q - q_o) \tag{7}$$

$$\left(\frac{1}{c_o}\right)\dot{P} = (g - q - L) P + \sqrt{P P_{in}} \tag{8}$$

where

$P = h\omega_o c S$, $P_{in} = h\omega_o c S_{in}$,

$g = \alpha n_l l/c$, $g_o = \alpha n_l^o l/c$,

$q = \beta n_a l/c$, $q_o = \beta n_a^o l/c$,

$L = \gamma_c l/c$ = cavity loss per pass,

$P_l = h\omega_o c[\gamma_l/2\alpha]$ = laser medium saturation power density,

and

$P_a = h\omega_o c[\gamma_a/2\beta]$ = absorber medium saturation power density.

Then the coupled equations (6)-(8) can be used to describe the performance of a passively Q-switched laser when injection locked.

III. Mean Output Power

Setting the time derivatives in (6)-(8) equal to zero, one gets the steady-state values of the gain g_s, absorber loss q_s, and mean power density P_s.

$$g_s = g_o/(1 + P_s/P_l) \tag{9}$$

$$q_s = q_o/(1 + P_s/P_a) \tag{10}$$

$$P_{in} = P_s \left(L + \frac{q_o}{1 + P_s/P_a} - \frac{g_o}{1 + P_s/P_l}\right)^2. \tag{11}$$

Equation (11) relates the mean power density with the injected power.

Fig. 2 shows that injection locking leads to a substantial increase in the mean output power. The parameter values of the experiment [8] for a CO_2 laser have been used. Table I gives the parameters for Nd:YAG and dye lasers [9]. The optimum transmittance T of the output mirror that yields maximum injection-locked power gain may be obtained by taking into account that the cavity loss per pass L is the sum of the residual loss L_i; and the useful mirror transmission T [5] and setting $\partial P_{out}/\partial T = 0$, where $P_{out} = TP_s$.

IV. Stability Analysis

A. *Determinantal Equation with an Injected Signal*

We shall follow the analysis of stability of passive Q-switching of single frequency lasers of Powell and Wolga [8]. In this analysis the assumed steady state is perturbed. The perturbation is taken to be a slow function of time. Thus, the gain, the absorber loss, and the optical power density are expressed in the form of small perturbation components x, y, and r around the steady-state values

$$g(t) = g_s + x(t),$$
$$q(t) = q_s + y(t),$$
$$p(t) = P_s + r(t). \tag{12}$$

Substituting (12) in (6)-(8), one gets

$$\dot{x} = -(g_s \gamma_l/P_l) r - x\gamma_l[(P_s/P_l) + 1] \tag{13}$$

$$\dot{y} = -(q_s \gamma_a/P_a) r - y\gamma_a[(P_s/P_a) + 1] \tag{14}$$

$$\dot{r} = \Omega_o \sqrt{(P_{in}/P_s)}\, r + 2\Omega_o P_s (x - y) \tag{15}$$

where

$$\Omega_o = \frac{c}{2l}.$$

Assuming the time dependence of x, y, and r to be proportional to exp (st), one gets a cubic determinantal equation

$$S^3 + a_1 S^2 + a_2 S + a_3 = 0 \tag{16}$$

where

$$a_1 = \gamma_l\left(\frac{P_s}{P_l} + 1\right) + \gamma_a\left(\frac{P_s}{P_a} + 1\right) + \Omega_o \sqrt{\frac{P_{in}}{P_s}} \tag{17}$$

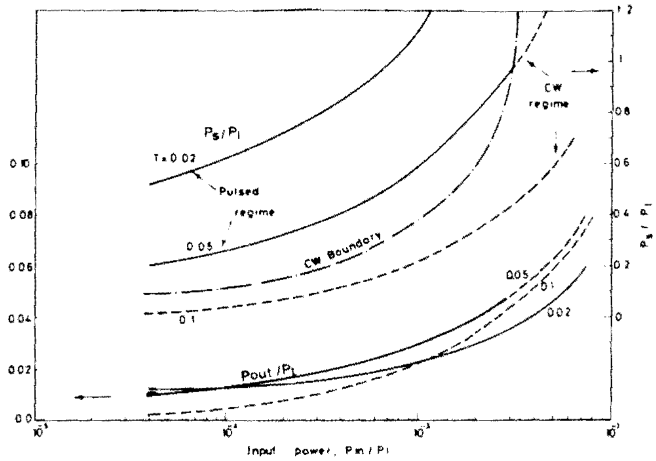

Fig. 2. Power output of a locked laser versus injected power. ($P_a/P_l = 2$, $g_o = 0.2$, $q_o = 0.1$, $L_i = 0.04$.)

$$a_2 = \gamma_l \gamma_a \left(\frac{P_s}{P_l} + 1\right)\left(\frac{P_s}{P_a} + 1\right) + 2\Omega_o P_s \left(\frac{\gamma_l g_s}{P_l} - \frac{\gamma_a q_s}{P_a}\right) + \Omega_o \left[\gamma_l \left(\frac{P_s}{P_l} + 1\right) + \gamma_a \left(\frac{P_s}{P_a} + 1\right)\right] \sqrt{\frac{P_{\text{in}}}{P_s}} \tag{18}$$

$$a_3 = 2\Omega_o P_s \gamma_a \gamma_l \left[\frac{q_s}{P_l}\left(\frac{P_s}{P_a} + 1\right) - \frac{q_s}{P_a}\left(\frac{P_s}{P_l} + 1\right)\right] + \gamma_l \gamma_a \Omega_o \left(\frac{P_s}{P_l} + 1\right)\left(\frac{P_s}{P_a} + 1\right)\sqrt{\frac{P_{\text{in}}}{P_s}}. \tag{19}$$

The necessary and sufficient conditions for stability (all three roots of (16) exhibit negative real part) are $a_1 > 0, a_3 > 0$, and $a_1 a_2 > a_3$ [8], [12]. Passive Q-switching results when a stability condition is not satisfied. If the relaxation time of the absorber is short compared to the pulse repetition period, one may consider the limit $\gamma_a \rightarrow \infty$ in (16)-(20). Then (16) reduces to a quadratic equation

$$S^2 + S\left\{\gamma_l\left(1 + \frac{P_s}{P_l}\right) - \frac{2\Omega_o q_o \frac{P_s}{P_a}}{\left(1 + \frac{P_s}{P_a}\right)} + \Omega_o \sqrt{\frac{P_{\text{in}}}{P_s}}\right\} + \Omega_o \gamma_l \left(\frac{2g_o \frac{P_s}{P_l}}{1 + \frac{P_s}{P_l}} - \frac{2q_o \frac{P_s}{P_a}\left(1 + \frac{P_s}{P_l}\right)}{\left(1 + \frac{P_s}{P_a}\right)^2} + \left(1 + \frac{P_s}{P_l}\right)\sqrt{\frac{P_{\text{in}}}{P_s}}\right) = 0. \tag{20}$$

With no injected signal, (20) is the same result as obtained by Haus [9]. It is to be noted that in the Haus analysis q_o and g_o are normalized by the cavity loss per pass L.

Instabilities result when either the S-independent term is negative, or the coefficient of S is negative.

TABLE I

	Nd: YAG	Dye Laser
P_l/P_a	0.13-1.3	3.7
γ_l	4×10^3 s^{-1}	2.5×10^8 s^{-1}
γ_a	10^{10}-10^{11} s^{-1}	10^9 s^{-1}
α/c	9×10^{-19} cm^2	10^{-16} cm^2
β/c	10^{-16} cm^2	3×10^{-16} cm^2

B. Pulse Repetition Rate and Energy

The pulse repetition rate of a passively Q-switched laser depends on the relaxation rates of both laser gain and absorber and cavity loss, as shown in Fig. 3. The energy per pulse E_o may be deduced as the product of the mean pulse power and the pulse repetition time.

The pulsation frequency may be approximated by the frequency of oscillation of small deviations about the equilibrium point [8]. The square of the oscillation frequency is given by a_2 for the cubic determinantal equation (16), and by the S-independent term of the quadratic equation (20), with a fast saturable absorber.

Fig. 4 shows that injection locking leads to a considerable increase in the output pulse energy and higher pulsation rate PRF than when free running. This may be physically explained as due to the reduction in the effective cavity loss.

C. Transition to CW Operation

A sufficiently strong injection CW signal results in a large circulating flux leading to the bleaching of the saturable absorber. In this case the repetitive Q-switching is quenched and the laser operates CW [7].

The maximum injected signal that allows pulsation may be deduced from the stability conditions of (16) or (20). CW operation results when all stability conditions are satisfied. With a fast absorber ($\gamma_a \rightarrow \infty$), CW operation requires that

$$\frac{2\Omega_o q_o \frac{P_s}{P_a}}{\left(1 + \frac{P_s}{P_a}\right)^2} \leqslant \gamma_l \left(1 + \frac{P_s}{P_l}\right) + \Omega_o \sqrt{\frac{P_{\text{in}}}{P_s}} \tag{21}$$

and

$$\frac{2q_o \frac{P_s}{P_a}}{\left(1 + \frac{P_s}{P_a}\right)^2} \leqslant \frac{2g_o \frac{P_s}{P_l}}{\left(1 + \frac{P_s}{P_l}\right)} + \sqrt{\frac{P_{\text{in}}}{P_s}}. \tag{22}$$

Transition between pulsed and CW operation would result when the stability conditions (22) is satisfied as an equality. The boundary of CW operation is plotted in Fig. 2 for $g_o = 0.2$,

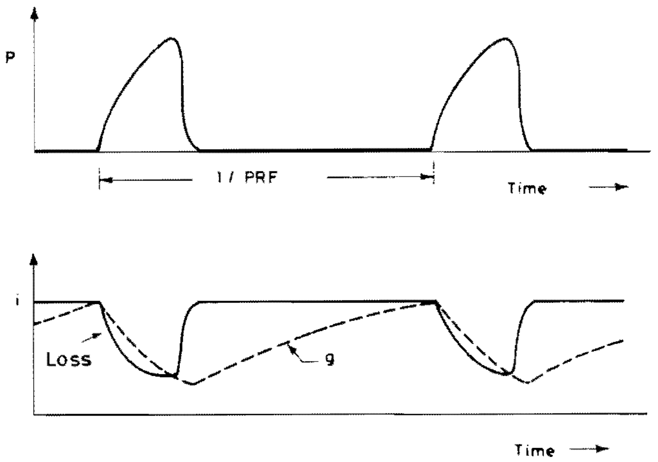

Fig. 3. Q-switch timing sequence.

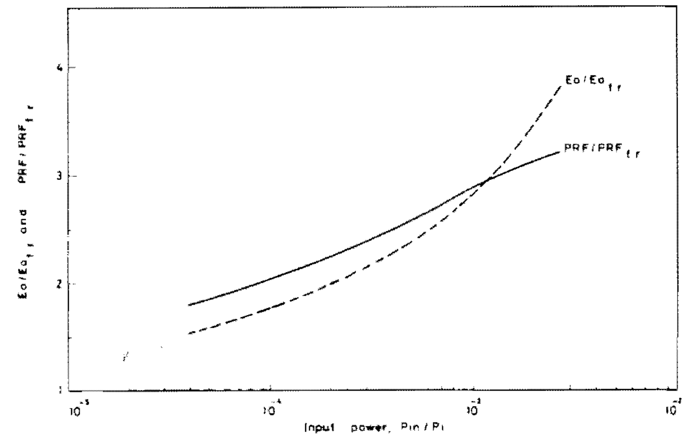

Fig. 4. The dependence of the PRF and pulse energy on the injected power. $P_a/P_l = 2, g_o = 0.2, q_o = 0.1, L_i = 0.04$, and $T = 0.05$. $PRF_{f.r.}$ and $E_{of.f.}$ correspond to free-running operation.

$q_o = 0.1$, $L_i = 0.04$, and $P_a = 2P_l$. Fig. 2 shows that with a 5 percent mirror transmittance, P_{in} should not exceed 0.06 P_{out} in order to maintain pulsed regime. Also, a proper selection of the mirror transmittance is necessary to ensure continued pulsed operation with an externally injected signal.

V. Conclusions

We presented a simplified mathematical model of three coupled rate equations for an injection-locked passively Q-switched laser. The model is applied to compute the average output power of an injection locked CO_2 laser.

The condition for CW laser operation is deduced from the stability analysis. It determines the maximum injection signal that allows Q-switching operation.

Further work is required to describe the capture of phase by the injection signal.

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